Efficient Perturbation Method for Computing Two-Port Parameter Changes due to Foreign Objects for WPT Systems

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The change in the two-port parameters of a wireless power transfer (WPT) chain are evaluated when an object (foreign object) with given material parameters is placed in the vicinity of the WPT transmitter and receiver. The change of the two-port parameters due to the presence of the foreign object has been derived by using the generalized theory of reciprocity. We emphasize that—instead of taking the differences of the two-port parameters calculated with and without the presence of the foreign object—the presented method provides the change of the two-port parameters directly from the calculated electromagnetic field at the location of the foreign object, as a result the relatively small parameter variations can be predicted with good accuracy.

***Index Terms*—foreign object detection, integral equation, reciprocity principle, wireless power transfer**

I. Introduction

Accurate and fast prediction of the interaction between a wireless power system (WPT) and a foreign object is frequently required during the design and operation of such devices. Usually the voltages and currents at the two-poles of the transmitter and receiver are measured to detect the foreign object. As the changes of the two-port values are very small compared to their nominal values, usually these changes are measured directly. The calculation method presented in this paper can compute the variation of the two-port characteristics directly, consequently it provides numerically robust and accurate method to predict the data used for the control of the WPT system.

In this paper first the method based on the general theory of reciprocity [1] is outlined. The original idea of such perturbation methods based on the reciprocity theory has been introduced for eddy-current nondestructive evaluation problems in [2] and has been further generalized for a wider range of low-frequency applications by [3]. Following this—based on the use of the Born approximation [4]—the possibility of evaluating the change of the two-port characteristics using only the calculation of the electromagnetic field without the presence of the foreign object is discussed. Finally, a simple numerical example demonstrates the method.

II. Configurations

Let us consider the two WPT configurations (Configurations A and B) shown in Fig. 1. Subscripts a and b are used to denote the electric, $\vec{E}$, and magnetic, $\vec{H}$, field components and material parameters (permeability, μ, permittivity, ε and conductivity, σ) associated to configurations A and B, respectively. The time variation of the electromagnetic (EM) field is considered to be the real part of $e^{j\omega t}$ and the complex notation is used to describe the steady-state field components. ω denotes the angular frequency of the excitation and t stands for the time.

In both configurations the WPT chain is embedded in linear material. The outer boundaries, Γ_0, of the investigated volume, Ω_0, is far from the WPT system and it extends well into the free space. The metallic components of the WPT coils are considered to be excluded from Ω_0.

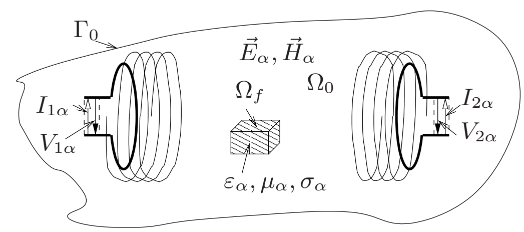

Fig. 1. The two investigated configurations. Configuation A: without the presence of the foreign object in volume Ω_f, $\alpha = a$. Configuration B: with the presence of the foreign object in volume Ω_f, $\alpha = b$.

The WPT chain is terminated by two-poles of which the currents and voltages are denoted by I_{1a}, I_{2a} and V_{1a}, V_{2a} (in configuration A) or I_{1b}, I_{2b} and V_{1b}, V_{2b} (in configuration B).

The two configurations are almost identical, there are only two exceptions: (i) the material properties of the two configurations in volume Ω_f that represents the foreign object are different (but outside of this volume they are the same for both cases), (ii) the feeding currents of the terminals I_{1a}, I_{2a}, I_{1b}, I_{2b} and the terminal voltages V_{1a}, V_{2a}, V_{1b}, V_{2b} are different for the two configurations, respectively.

III. Calculation of the two-port parameters

Based on the generalized concept of reciprocity [1], the relation between the port voltages and currents of the two configurations can be derived. Skipping the detailed derivation (this will be presented in the full version) we arrive at the following equation:

$$\begin{aligned} I_{1b}V_{1a} - I_{1a}V_{1b} + I_{2b}V_{2a} - I_{2a}V_{2b} \\ = -\int_{\Omega_f} j\omega(\mu_b - \mu_a)\vec{H}_a \cdot \vec{H}_b \, d\Omega \\ + \int_{\Omega_f} \left[(j\omega\varepsilon_b + \sigma_b) - (j\omega\varepsilon_a + \sigma_a)\right] \vec{E}_a \cdot \vec{E}_b \, d\Omega. \quad (1) \end{aligned}$$

The two-port impedance characteristics associated with the configurations A and B can be written as:

$$V_{1a} = Z_{11}I_{1a} + Z_{12}I_{2a}, \tag{2}$$
$$V_{2a} = Z_{21}I_{1a} + Z_{22}I_{2a}, \tag{3}$$
$$V_{1b} = (Z_{11} + \Delta Z_{11})I_{1b} + (Z_{12} + \Delta Z_{12})I_{2b}, \tag{4}$$
$$V_{2b} = (Z_{21} + \Delta Z_{21})I_{1b} + (Z_{22} + \Delta Z_{22})I_{2b}, \tag{5}$$

where Z_{11}, Z_{12}, Z_{21} and Z_{22} are the impedance parameters associated with the two-port describing the WPT chain without the presence of the foreign object. ΔZ_{11}, ΔZ_{12}, ΔZ_{21} and ΔZ_{22} are the changes of the associated impedance parameters due to the presence of the foreign object. Our goal is to find a robust numerical method for the determination of these changes. Note that the reciprocity of the arrangement assures that $Z_{12} \equiv Z_{21}$ and $\Delta Z_{12} \equiv \Delta Z_{21}$.

Let us multiply (2)-(5) with I_{1b}, I_{2b}, I_{1a}, I_{2a}, respectively and plug this into (1),

$$\begin{aligned}\Delta Z_{11}I_{1a}I_{1b} + \Delta Z_{22}I_{2a}I_{2b} + \Delta Z_{12}(I_{1a}I_{2b} + I_{1b}I_{2a}) \\ = \int_{\Omega_f} j\omega(\mu_b - \mu_a)\vec{H}_a \cdot \vec{H}_b \, d\Omega \\ - \int_{\Omega_f} \left[(j\omega\varepsilon_b + \sigma_b) - (j\omega\varepsilon_a + \sigma_a)\right] \vec{E}_a \cdot \vec{E}_b \, d\Omega.\end{aligned} \tag{6}$$

From this expression the impedance variation can be directly calculated. The actual ΔZ_{11}, ΔZ_{22} and ΔZ_{12} values can be obtained by, e.g., evaluating the integrals in (6) for 3 linearly independent combinations of the currents, then the resulting linear system can be solved for the impedance variations.

IV. Born approximation

For the sake of simplicity let us assume that $\sigma_a = \sigma_b$ and $\mu_a = \mu_b$. In such situations, based on the first order Born approximation [4], $\vec{E}_b$ in (6) can be approximated by $\vec{E}_a$ if the permittivity of volume Ω_f in Configuration B is close to the one in the case of Configuration A and if the same port currents are used for Configurations A and B, i.e., $I_{1a} = I_{1b} = I_1$ and $I_{2a} = I_{2b} = I_2$. For this situation one can write

$$\Delta Z_{11}I_1^2 + \Delta Z_{22}I_2^2 + 2\Delta Z_{12}I_1I_2 \approx -\int_{\Omega_f} j\omega\left(\varepsilon_b - \varepsilon_a\right)\vec{E}_a^2 \, d\Omega. \tag{7}$$

Consequently, we can conclude that—by knowing the electric field, $\vec{E}_a$, in the volume Ω_f calculated without the presence of the foreign object—one can get an approximation of the change of the impedance parameters due to the presence of the foreign object by evaluating the integral (7). This can be done if the conditions necessary for the validity of the first order Born approximation are satisfied.

If the permittivity differences between the configurations are larger, one may use higher order Born approximations [4] to get a good prediction of the impedance parameter changes based on $\vec{E}_a$. As a result, the impedance variation and the $\vec{E}_a$ field can be linked together also for larger permittivity differences. In the full version of the paper we will investigate the conditions needed for the application of the first and higher order Born approximations and compare these results with the ones obtained by alternative standard methods.

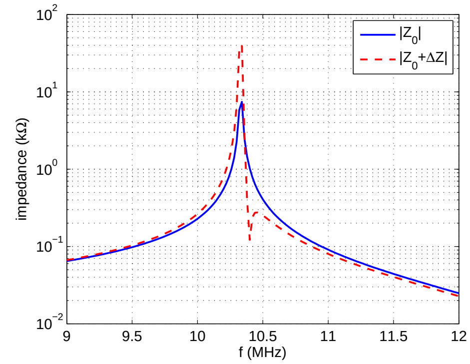

Fig. 2. Absolute value of the input impedances of the configurations with $(Z_0 + \Delta Z)$ and without (Z_0) the presence of the foreign object.

The impedance variation due to the presence of a conducting and/or magnetic foreign object can be obtained by the Born approximation in a similar way starting from (6) if the necessary conditions hold.

V. Numerical example

In a simple numerical example we investigate the variation of the input impedance of a WPT transmitter that is a helical coil with radius 30 cm and height 20 cm, having 5.25 turns and being excited via a matching loop with radius 25 cm. The base plane of the helix is parallel with the loop and they are in coaxial alignment. All wires have a radius of 3 mm and they are made of copper. The coil terminals are open and the loop is connected to an ac current source. The effect of a dielectric sphere (i.e., the foreign object) with a relative permittivity $\varepsilon_r = 1.5$ is studied. The sphere has a radius of 25 cm and it is located at the middle of the helix.

The input impedance of the loop (Z_0) is calculated by the method of moments [5] without the sphere, and the change of the input impedance (ΔZ_0) is determined by the 1st order Born approximation as outlined above. The results are presented in Fig. 2. The impedance curve seems to be slightly shifted downwards that can be explained by the stronger capacitive effects caused by the sphere.

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